Appearance of the secular terms in the pertubative expantion in an unharmonic oscillation

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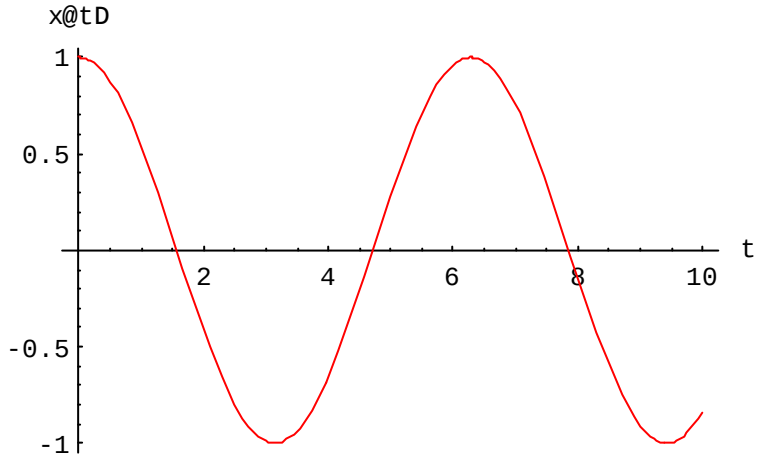
Harmonic Oscillation

$$x''(t) + x(t) = 0$$

```
harmsol = NDSolve@8x''@tD + x@tD Š 0, x@0D Š 1, x'@0D Š 0<, x@tD, 8t, 0, 100<D;
```

```
harm = Plot@x@tD •. harmsol@@1DD, 8t, 0, 10<, PlotStyle ® 8RGBColor@1, 0, 0D<, AxesLabel ® 8t, x@tD<D
```




```
… Graphics …
```

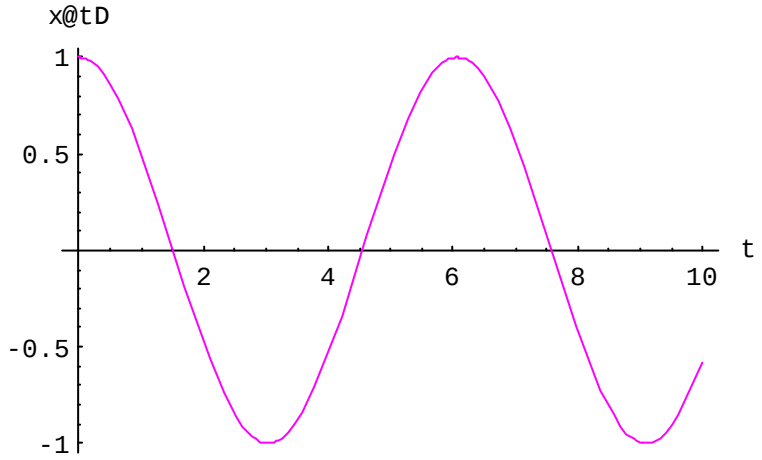
Unharmonic Oscillation (numerical solution)

$$x''(t) + x(t) = -e\, x(t)^3$$

For e=1/10

```
unhsol = NDSolve@8x''@tD + x@tD Š -e x@tD^3, x@0D == 1, x'@0D Š 0< •. e ® 1 • 10 , x@tD, 8t, 0, 100<D;
```

```
unharm = Plot@x@tD •. unhsol, 8t, 0, 10<, PlotStyle ® 8RGBColor@1, 0, 1D<, AxesLabel ® 8t, x@tD<D
```




```
… Graphics …
```

Unharmonic Oscillation (perturbative solution)

$$x''(t) + x(t) = -\epsilon\, x(t)^3$$

The perturbative solution is

```
xpert@t_D := A Cos@tD + e H A^3 • 32 Cos@3 tD - A^3 3 • 8 t Sin@tDL; xpert@tD
```

$$A\,\text{Cos}[t] + e\left(\frac{1}{32}A^3\,\text{Cos}[3\,t] - \frac{3}{8}A^3\,t\,\text{Sin}[t]\right)$$

where the amplitude A is fixed by the initial condition

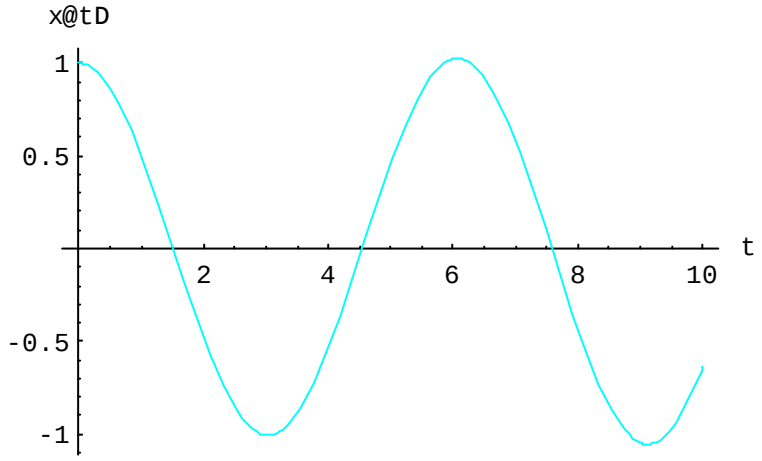
```
amp = NSolve@xpert@0D Š 1 , AD;
```

For e=1/10

```
N@amp •. e ® 1 • 10 D
```

```
88A ® -0.498452 - 17.9094 ä<, 8A ® -0.498452 + 17.9094 ä<, 8A ® 0.996904<<
```

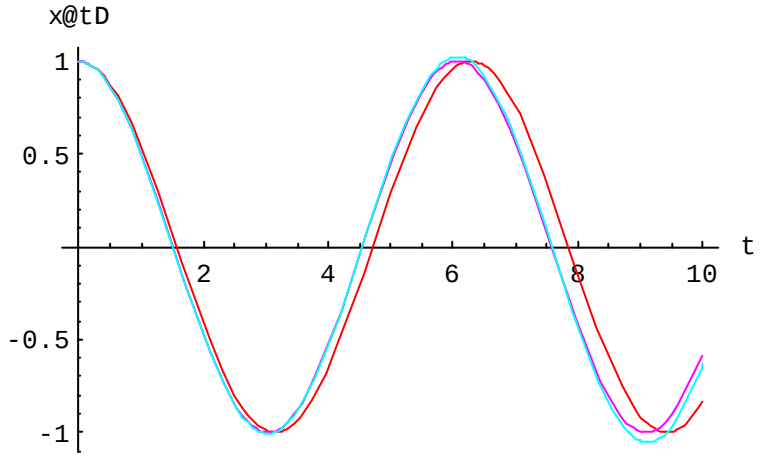
```
pert = Plot@xpert@tD /. amp@@3DD /. e ® 1 • 10 , 8t, 0, 10<, PlotStyle ® 8RGBColor@0, 1, 1D<, AxesLabel ® 8t, x@tD<D
```




```
… Graphics …
```

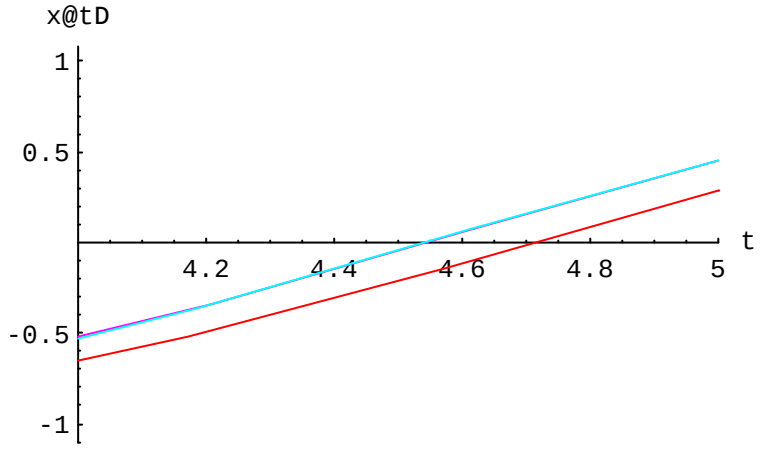
Comparison

```
Show@harm, unharm, pertD
```




```
… Graphics …
```

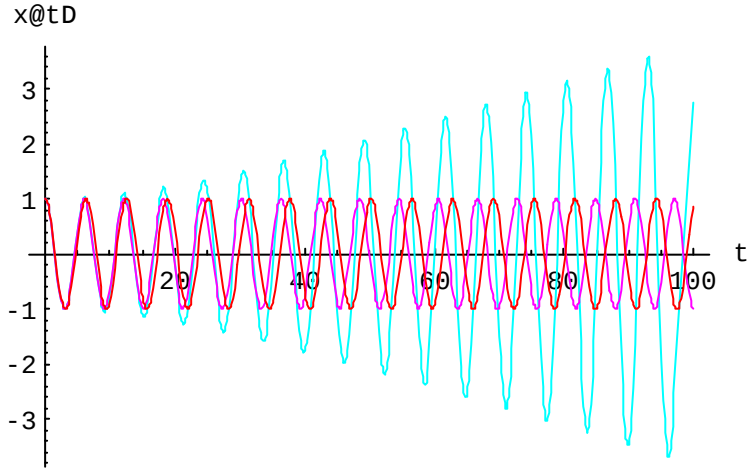
```
Show@harm, unharm, pert, PlotRange ® 884, 5<, Automatic<D
```




```
… Graphics …
```

Large t behavior

```
Plot@8xpert@tD •. amp@@3DD •. e ® 1 • 10 , x@tD •. unhsol ,
  x@tD •. harmsol<, 8t, 0, 100<,
 PlotStyle ® 8RGBColor@0, 1, 1D, RGBColor@1, 0, 1D, RGBColor@1, 0, 0D<, AxesLabel ® 8t, x@tD<D
```

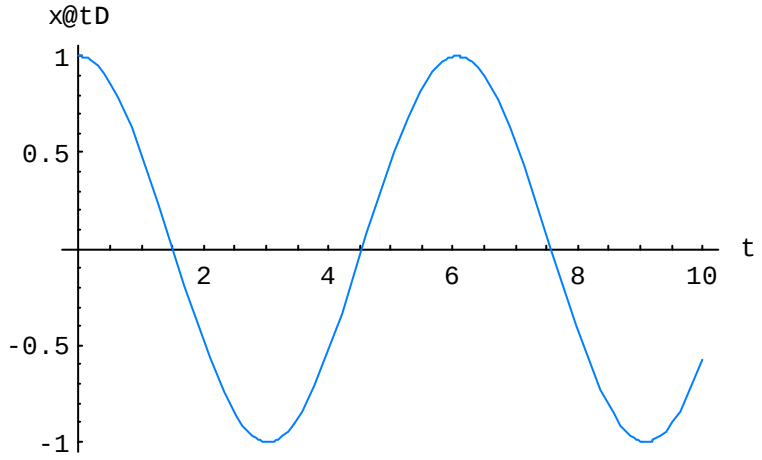



```
… Graphics …
```

Unharmonic Oscillation (Poincare-Lindstedt Expansion)

```
xpl@t_D := A Cos@ H1 + 3 A^2 • 8 eL tD + e A^3 • 32 Cos@ 3 H1 + 3 A^2 • 8 e L tD;
```

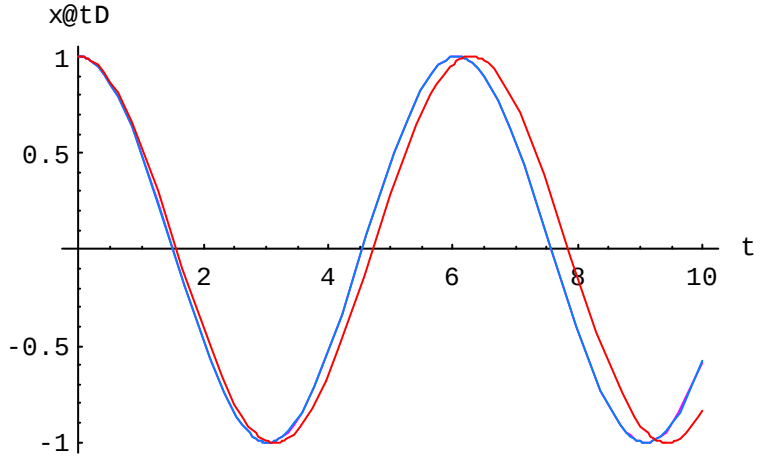
```
pl = Plot@xpl@tD •. amp@@3DD •. e ® 1 • 10 , 8t, 0, 10<, PlotStyle ® 8RGBColor@0, 0.5, 1D<, AxesLabel ® 8t, x@tD<D
```




```
… Graphics …
```

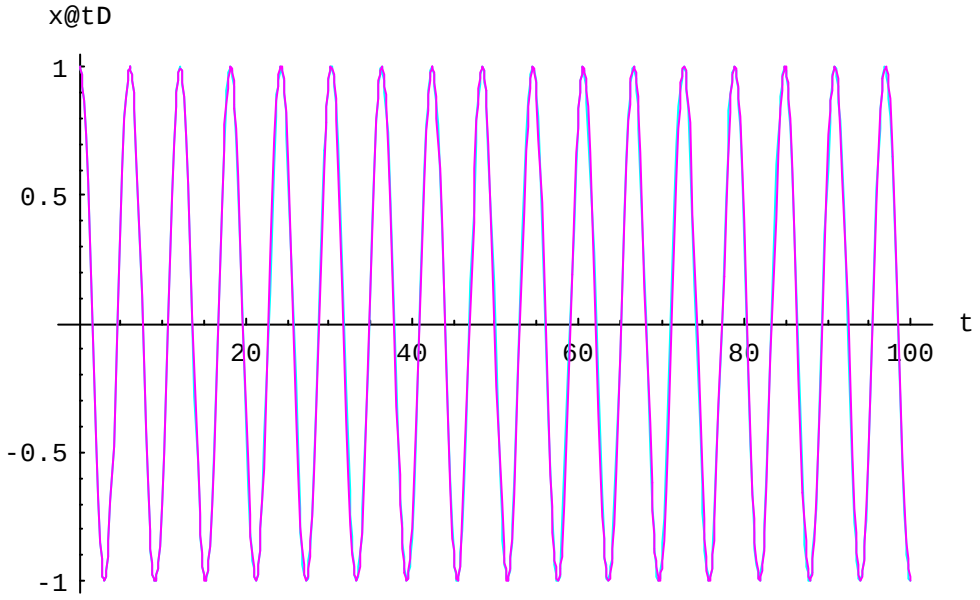
Comparison 2

```
Show@unharm, pl, harmD
```




```
… Graphics …
```

```
Plot@8xpl@tD •. amp@@3DD •. e ® 1 • 10 , x@tD •. unhsol<, 8t, 0, 100<,
 PlotStyle ® 8RGBColor@0, 1, 1D, RGBColor@1, 0, 1D<, AxesLabel ® 8t, x@tD<D
```




```
… Graphics …
```