微積分♪演習 (略解) (情報メディア学科 1 年次科目)

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8 方向微分と多変数関数の合成微分

8.1 お奨め問題: 多変数関数の合成微分

略解

1. 上で求めたように, $\frac{\partial f}{\partial x}(x,y) = 2x$, $\frac{\partial f}{\partial y}(x,y) = 1$. $\frac{\mathrm{d}z}{\mathrm{d}t}(t) = (2\xi(t)) \cdot (-\sin t) + 1 \cdot (\cos t) = -2\cos t \sin t + \cos t$.
2. $\xi(0) = 1, \eta(0) = 0$, $\frac{\partial \xi}{\partial t}(0) = 0$, $\frac{\partial \eta}{\partial t}(0) = 0$. $\frac{\mathrm{d}z}{\mathrm{d}t}(0) = (2\xi(0)\mathrm{e}^{\xi(0)+\eta(0)} + \xi(0)^2\mathrm{e}^{\xi(0)+\eta(0)})\frac{\partial \xi}{\partial t}(0) + (\xi(0)^2\mathrm{e}^{\xi(0)+\eta(0)})\frac{\partial \eta}{\partial t}(0) = \mathrm{e}^1$.
3. $r(\frac{\sqrt{3}}{2}, \frac{1}{2}) = 1$. $\frac{\partial r}{\partial x}(x,y) = \frac{x}{\sqrt{x^2+y^2}}$. $\frac{\partial r}{\partial x}(\frac{\sqrt{3}}{2}, \frac{1}{2}) = \frac{\sqrt{3}}{2}$. $\frac{\partial g}{\partial x}(x,y) = \frac{\mathrm{d}g}{\mathrm{d}r}(r(x,y)) \cdot \frac{\partial r}{\partial x}(x,y)$. $\frac{\partial g}{\partial x}(\frac{\sqrt{3}}{2}, \frac{1}{2}) = \frac{\mathrm{d}g}{\mathrm{d}r}(1) \cdot \frac{\partial r}{\partial x}(\frac{\sqrt{3}}{2}, \frac{1}{2}) = 22 \cdot \frac{\sqrt{3}}{2} = 11\sqrt{3}$.

8.2 方向微分

略解

1.
2. $\frac{\partial f}{\partial x}(x,y) = 2x$, $\frac{\partial f}{\partial x}(-1,1) = -2$, $\frac{\partial f}{\partial y}(x,y) = 1$, $\frac{\partial f}{\partial y}(-1,1) = 1$.
3. $z - 2 = -2(x+1) + (y-1)$.
4. $D_{\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} f(-1,1) = -\frac{1}{2}(-2) + (+\frac{\sqrt{3}}{2}) \cdot 1 = +1 + \frac{\sqrt{3}}{2}$.

   または直接に $\frac{\mathrm{d}}{\mathrm{d}t}\{(-1-\frac{1}{2}t)^2 + (1+\frac{\sqrt{3}}{2}t)\}$ の $t = 0$ における値を計算してもよい.

8.3 チャレンジ問題: 2 変数関数のグラフ

次の関数 $f(x,y)$ について, 等高線プロットを描こう. 3 次元プロット (鳥瞰図) を想像しよう (絵心のある人は描こう).

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略解

1. 等高線は楕円. 別紙.
2. 等高線は双曲線. 別紙.
3. 等高線は $y = C\mathrm{e}^{-x}$. 別紙.

2006.8.2.1

2005

7.1.1

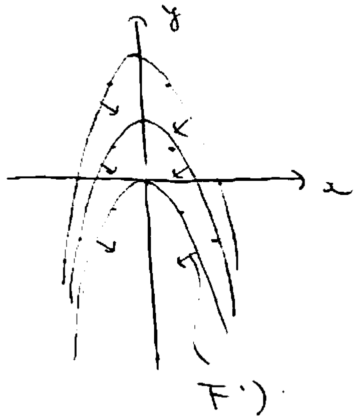
2004

8.1.1.

$f(x,y) = x^2 + y$

$x^2 + y = C$

$y = -x^2 + C$ （放物線）



2006. 8.3
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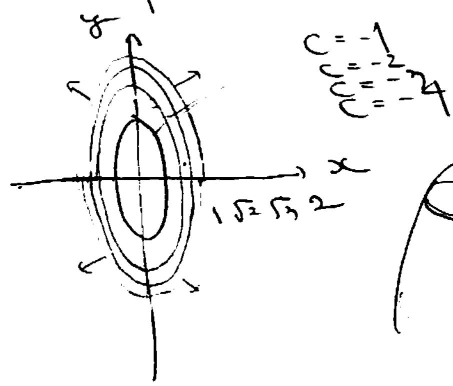
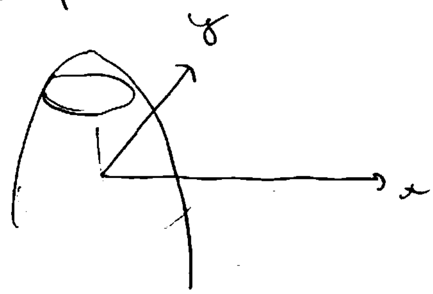
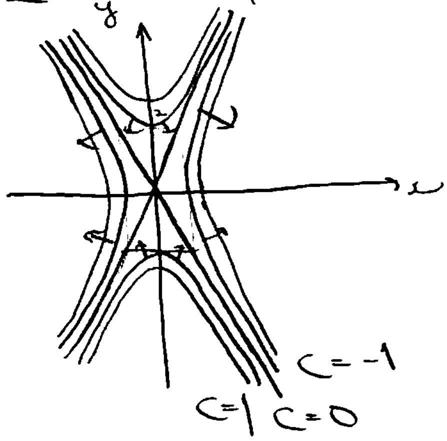
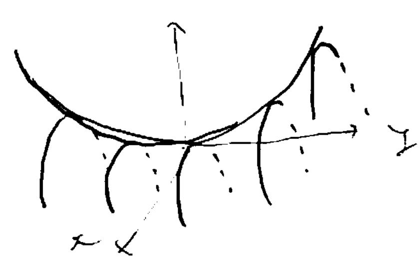
1 $-x^2 - \frac{1}{4}y^2 = C$





2 $-x^2 + \frac{1}{4}y^2 = C$





3 $ye^x = C$

$y = Ce^{-x}$